**4.3**

**13.** (4k + 3)² - 1 = 16k² + 24k + 9 – 1 = 8(2k² + 3k + 1)

Since (2k² + 3k + 1) is an integer and k is an integer; the sums and products of integers are also integers. Therefore, this equation will always be a multiple of 8 and divisible by 8.

**34.** Let P = Pennies, D = Dimes and Q = Quarters and must be integers. Also satisfy two equations:

1. P + 10D + 25Q = 300
2. P + D + Q = 50

Starting with Q=12, 25\*12 = 300, and 38 coins remaining. Q = 2D + 5P, and a net change of 6 coins. Since 38 is not divisible by 6 there is no solution with this equation.

With D=30, 10\*30 = 300, and 20 coins remaining. D = 10P and a net change of 9 coins. Since 20 is not divisible by 9 there is no solution with this equation.

Other possible net changes:

Q to P = +24 coins

P to D = -9 coins

P to Q = -24 coins

All multiples of net change are 3 coins and we require a net change of 38 or 20 coins which are not divisible by 3. Therefore there is no solution.

**4.4**

**19.** Proof:

Suppose n is any integer. By the quotient remainder theorem with d = 2, n = 2q is even or n = 2q + 1 is odd.

*Case 1 (n is even):* n = 2q for some integer q.

n² - n + 3

= (2q) ² - 2q + 3 by substitution, let n = 2q

= 4q² - 2q + 2 + 1

= 2(2q² - q + 1) + 1 by algebra

Let x = (2q² - q + 1). Then x is an integer since products, sums and differences of integers are integers. Thus, substituting,

n² - n + 3 = 2x + 1 definition of odd

*Case 2 (n is odd):* n = 2q + 1 for some integer q.

n² - n + 3

= (2q + 1) ² - 2q + 1 + 3 by substitution, let n = 2q + 1

= 4q² + 4q - 2q + 2 + 3

= 4q² + 2q + 4 + 1

= 2(2q² + q + 2) + 1 by algebra

Let x = (2q² + q + 2). Then x is an integer since products and sums of integers are integers. Thus, substituting,

n² - n + 3 = 2x + 1 definition of odd

Thus for all integers n, n² - n + 3 is an odd integer.

□

**4.5**

**9.** Floor is more appropriate. If ceiling is used we have to subtract a box when we have a non-integer from n/36.

Using floor notation:

If n/36 is an integer, then the number of boxes = floor (n/36)

If n/36 is not an integer, then the number of boxes = floor (n/36)

**4.6**

**12.** Proof:

Suppose not. Suppose that a + br is rational.

a + br = c

m, n, p, q, s, t ∈ ℤ, n, q, t ≠ 0 (definition of rationalism)

by substitution

by algebra

by algebra

Since b ≠ 0, p ≠ 0. Thus since, p, n, t ≠ 0 we have implied r is a rational number, which is a contradiction. Therefore, supposition is false and the original statement is true.

□

**4.7**

**12.** False

Let √2 be the irrational number.

√2 ∙ √2 = 2, which is rational.

**4.8**

4,131 and 2,431

2431/4131 gives remainder of 1700, with quotient of 1.

1700/2431 gives remainder of 731 with quotient of 1.

731/1700 gives remainder of 238 with quotient of 2.

238/738 with remainder 17, quotient 3.

17/238 with remainder 0! Yay! Quotient 14.

Now we add all quotients which gives

17 as the GCD